Similar Laminar Boundary Layers with Zero Wall Shear and Mass Addition

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The compressible similar laminar boundary-layer equations for unit Prandtl number are considered for the case of zero wall shear and values of mass addition, such that blowoff phenomena are observed. An analytical technique based on asymptotic methods is used to describe the solutions in a regime where numerical computation fails. It is shown for the limit of small adverse pressure gradients that solutions in the wall region which grow exponentially with distance from the wall can be matched to a mixing-layer (shear-layer) solution by means of a purely translational asymptotic transformation relating the wall and shear-layer variables. The results indicate a thickening of the boundary layer proportional to the logarithm of the small pressure gradient for finite wall enthalpy; whereas for zero wall enthalpy the rate is inversely proportional to the pressure gradient. Results are given for the mass addition rate and the wall heat transfer as a function of the prescribed small adverse pressure gradient for a range of wall temperatures. Pathelogical results for heat transfer rates found for the cold wall boundary condition in earlier numerical calculations are rationalized on the basis of the analytical results.

1. Introduction

In a recent paper, Fox and Saland[1] presented the compendium of solutions for the compressible similar laminar boundary-layer equations when the wall shear is identically zero. Numerical computation was used for a large spectrum of wall enthalpy and injection (suction) boundary conditions to develop solutions which include pressure gradient parameter values compatible with the given system and wall heat-transfer rates. Among the problems considered were a class involving mass addition in which incipient blowoff phenomena was observed to occur. That is to say, the region of strong viscous effects ordinarily near the wall moved into the interior of the fluid, causing a dramatic extension of the overall boundary-layer dimension, with a concomitant reduction in thermal and velocity gradients in the neighborhood of the surface. Thus, numerical injection results in Ref. 1 are cited for values of the injection parameter $-f_\omega$, at most equal to 0.85 (and in some cases involving cold wall conditions, at most 0.80). Numerical integration by means of the trapezoidal rule used in Ref. 1 for the remaining range up to the classical blowoff value $-f_\omega = C_o = 0.87574$[2,3] was apparently unsuccessful due to the presence of extremely small gradients and the largeness of the field.†

In addition to the more prosaic blowoff phenomena, Fox and Saland obtained rather peculiar (not necessarily erroneous) results for the cold wall boundary condition, $g_\omega \to 0$. The numerical calculation apparently did not converge for values of $-f_\omega > 0.8$ implying that the gradients became smaller faster than when $g_\omega = 0(1)$. As an example of the odd behavior, a nonmonotonic variation in heat transfer rate with decreasing wall enthalpy was observed. For a given value of injection $-f_\omega$ greater than about 0.60, $g'(0)$ first increased as the prescribed wall enthalpy decreased from the external stream value. However, for values $g_\omega \lesssim 0.2$ the heat-transfer rate decreased as $g_\omega \to 0$. No explanation was offered for this reversal.

Incipient blowoff phenomena is of some interest to the technology because it is associated with large reductions in heat transfer. Hence, it is worthwhile to consider in some detail the structure of the flow configuration under conditions for which numerical computations fail. In the present paper an asymptotic analysis is used to describe the singular behavior alluded to in Ref. 1. Using a method which has been developed in several earlier papers,[3–5] the approach to incipient blowoff is described. Of particular interest are the functional descriptions of the heat-transfer reduction and boundary-layer growth. It is also shown that the cold wall boundary condition $g_\omega = 0$ leads to somewhat peculiar results which, however, verify the trends indicated in Ref. 1. An explanation of the behavior is offered.

2. Describing Equations

The describing equations for compressible similar laminar flow with unit Prandtl number and zero wall shear are

$$L(f) = f''' + ff'' + \hat{\beta}(g - f'^2) = 0 \tag{1a}$$

$$M(g) = g'' + fg' = 0 \tag{1b}$$

$$f' = f'' = 0, g = g_\omega \text{ at } \eta = 0 \tag{1c}$$

$$f' = g = 1 \quad \text{for } \eta \to \infty \tag{1d}$$

where η is the usual similarity variable, f the reduced stream function, and g the enthalpy. This fifth-order system, which is well posed, can be solved for prescribed values of the pressure gradient parameter $\hat{\beta}$. One may surmise from the results of Ref. 1 that in order to develop solutions of Eq. 1, which describe incipient blowoff phenomena, the pressure gradient parameter $\hat{\beta}$ must be small and negative and the injection rate $f(0) = f_\omega$ nearly equal to $-C_o = -0.87574\ldots$. Such asymptotic behavior can be verified formally by considering solutions to Eq. (1) in terms of asymptotic expansions based on the limit $\beta = -\hat{\beta} \to 0^+$ for which $f(0) = f_\omega \to -C_o$. The analysis is developed in terms of a matched asymptotic scheme in which the inner (or wall) region is described by expansions based on the limit process $\beta \to 0$, η fixed. The appropriate independent variable in the outer (or shear) layer

Received July 17, 1970; revision received October 15, 1970. This research was partially supported by a Summer Research Initiation Faculty Fellowship at the University of Colorado. The author is indebted to P. A. Libby, who provided several cogent remarks in the early stages.

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† Rogers[10,11] has shown that related calculations can be carried out by using a fourth-order Runge-Kutta integration with a Nachtsheim-Swigert iteration scheme for cases other than the so-called "cold wall."

is defined by the purely translational transformation,

$$z = \eta - C_o^{-1}\left[\ln\lambda_o(\beta) + \ln\lambda_1 + \sum_{n=2} \lambda_n(\beta)\right] \quad (2)$$

where $\lambda_o(\beta)$ is an asymptotically large algebraic function of β, λ_1 is a constant, and λ_n, $n \geq 2$ are asymptotically small functions of β; all of which are found in the process of analysis. The outer expansions are based on the limit process $\beta \to 0$, z fixed. This form of transformation used in several related papers,[3–5] can be interpreted to mean that the outer part of the wall layer is defined by $\eta \to \infty$, whereas one approaches the wall from the shear layer when $z \to -\infty$.

Furthermore, the line $z = 0$ describes the location of the dividing streamline which is embedded in the shear layer. Since the wall region is generally thicker than the shear layer, the expression

$$\eta^* = C_o^{-1}\left(\ln\lambda_o + \ln\lambda_1 + \sum_{n=0} \lambda_n\right)$$

can be used as a measure of the characteristic dimension of the total boundary-layer thickness. Finally, the matching relation between the wall and shear-layer expansions can be written such that there will be a term by term comparison of the two expansions when the asymptotic form of the outer ($z \to -\infty$) is written in terms of η and then related to the inner.

3. Asymptotic Analysis

Rather than describing the actual chronological construction of the asymptotic sequences in the expansions, it is perhaps more useful for explanatory purposes to simply state their values, develop the hierarchy of systems which describe several orders of the expansions, and then show that the solutions satisfy the relevant boundary and matching conditions. This procedure will simplify the discussion considerably, particularly with regard to the presence of so-called "switchback" terms[6] which produce a certain degree of complexity in the matching technique.

Wall-Layer Solution

It is asserted that near the wall the expansions for the dependent variables may be written in the form

$$f \sim -C_o + \beta \ln^2(1/\beta)f_1(\eta) + \beta \ln(1/\beta)f_2(\eta) + \beta f_3(\eta) + 0(\beta^2 \ln^3\beta) \quad (3a)$$

$$g \sim g_o(\eta) + \beta \ln^2(1/\beta)g_1(\eta) + \beta \ln(1/\beta)g_2(\eta) + \beta g_3(\eta) + 0(\beta^2 \ln^3\beta) \quad (3b)$$

where it is to be noted that $f_o = -C_o$. (The validity of the latter assumption may be inferred from Kassoy's[8] blowoff analysis, the numerical results of Ref. 1, and/or a posteriori from the present work.) Substitution of the expansions into Eqs. (1a–1c) and gathering of like order terms leads to the hierarchy of systems:

$$f_o = -C_o,\ g_o'' - C_o g_o' = 0,\ g_o(0) = g_\omega$$

$$f_m''' - C_o f_m'' = 0,\ f_m'(0) = f_m''(0) = 0$$

$$g_m'' - C_o g_m' = 0,\ g_m(0) = 0, \text{ for } m = 1,2 \text{ and}$$

$$f_3''' - C_o f_3'' = g_\omega,\ f_3'(0) = f_3''(0) = 0$$

$$g_3'' - C_o g_3' = 0,\ g_3(0) = 0$$

which have the elementary solutions

$$g_o = g_\omega + K_o(e^{C_o\eta} - 1) \quad (4a)$$

$$g_m = K_m(e^{C_o\eta} - 1),\ m = 1,2,3 \quad (4b)$$

$$f_m = B_m,\ m = 1,2 \quad (4c)$$

$$f_3 = (g_\omega/C_o^3)e^{C_o\eta} - (g_\omega/2C_o)\eta^2 - (g_\omega/C_o^2)\eta + B_3 \quad (4d)$$

The undetermined constants K_o, K_m, B_m, B_3 must be determined through use of the matching condition, which, it should be noticed, involves terms of growing exponentially in η. Similar forms have been encountered in Ref. 3.

Shear-Layer Solution

The relevant outer equations, constructed by using $F(z) = f(\eta)$, $G(z) = g(\eta)$, $\beta = -\hat{\beta}$ and Eq. (2) in Eqs. (1a), (1b), and (1d) are

$$L(F) = 0 \quad \textbf{(1a')}$$

$$M(G) = 0 \quad (1b')$$

$$F' = G = 1,\ z \to \infty;\quad F = 0,\ z = 0 \quad (1c')$$

where the last boundary condition defines the dividing streamline. Solutions to this system are developed in terms of expansions of the assumed form:

$$F \sim F_o(z) + \beta \ln(1/\beta)F_1(z) + \beta F_2(z) + 0(\beta^2 \ln^3\beta) \quad (5a)$$

$$G \sim G_o(z) + \beta \ln(1/\beta)G_1(z) + \beta G_2(z) + 0(\beta^2 \ln^3\beta) \quad (5b)$$

which may be substituted into Eq. (1′) to produce the systems

$$F_o''' + F_oF_o'' = 0;\quad F_o'(\infty) = 1,\ F_o(0) = 0 \quad (6a)$$

$$G_o'' + F_oG_o' = 0;\quad G_o(\infty) = 1 \quad (6b)$$

$$F_1''' + F_oF_1'' + F_o''F_1 = 0;\quad F_1'(\infty) = F_1(0) = 0 \quad (7a)$$

$$G_1'' + F_oG_1' = -F_1G_o;\quad G_1(\infty) = 0 \quad (7b)$$

$$F_2''' + F_oF_2'' + F_o''F_2 = G_o - F_o'^2 \qquad F_2'(\infty) = F_2(0) = 0 \quad (8a)$$

$$G_2'' + F_oG_2' = -F_2G_o';\quad G_2(\infty) = 0 \quad (8b)$$

The lowest order stream function, described by a Blasius equation, must satisfy the usual external flow condition, a dividing streamline condition, and the preliminary matching condition, $F_o(z \to -\infty) \sim -C_o$ derived from Eqs. (2), (3a), and (5a). Hence, the complete system for F_o describes Lock's[7] mixing layer. One finds the complete solution numerically,[2] and the asymptotic form[3]:

$$F_o(z \to -\infty) \sim -C_o + ae^{C_oz} + 0(e^{2C_oz}) \quad (9)$$

where $a = 1.1502$.

The G_o solution, obtained without further effort from the Crocco type relationship $G_o = G_o(-\infty) + [1 - G_o(-\infty)]F_o'$ where $G_o(-\infty)$ is a constant to be found, has the asymptotic form

$$G_o(z \to -\infty) \sim G_o(-\infty) + [1 - G_o(-\infty)]aC_oe^{C_oz} + 0(e^{2C_oz}) \quad (10)$$

which may be combined with Eqs. (2, 3b, 4a, and 5b) to obtain the matching condition

$$G_o(-\infty) + [1 - G_o(-\infty)]\frac{(aC_o/\lambda_1)}{\lambda_o(\beta)}e^{C_o\eta} + \ldots \sim g_\omega + K_o(e^{C_o\eta} - 1) + 0(\beta^2 \ln 1/\beta) \quad (11)$$

Since all terms beyond the first on the left-hand side of Eq. (11) are asymptotically small with respect to the parameter β, it follows that $K_o = 0$, and $G_o(-\infty) = g_\omega$.

Stewartson[8] has discussed the solution to the first-order stream-function system in Eq. (7a) which can be written in quadrature form

$$F_1 = A_{11}\left[F_o'(z)\int_o^z \frac{F_o''(\tau F_o' + F_o)d\tau}{(2F_o'^2 + F_o''')^2} + (zF_o' + F_o)\int_z^\infty \frac{F_o'F_o''d\tau}{(2F_o'^2 + F_o''')^2}\right]$$

where τ is a dummy integration variable and A_{11} an integration constant.

The asymptotic form is

$$F_1(z \to -\infty) = A_{11}\left(-\frac{1}{C_o^2}\left\{z + \hat{z} + \left[-\frac{az^2}{2} + a\left(\frac{2}{C_o} - \hat{z}\right)z + 0(1)\right]e^{C_o z} + \ldots\right\}\right) \quad (12)$$

where $\hat{z} = 2.337$.

Once again the corresponding enthalpy solution can be obtained from the Crocco-like relation $G_1 = C_1(1 - F_o') + (1 - g_\omega)F_1'$ which satisfies the system in Eq. (7b). The integration constant is found from the matching procedure.

The asymptotic form of G_1, necessary for use in the matching condition, can be constructed from the Crocco-relation and Eqs. (9) and (12) in the form

$$G_1(z \to -\infty) = G_1(-\infty) + A_{11}(1 - g_\omega)(a/2C_o) \times [z^2 + 2(\hat{z} - 1/C_o)z + 0(1)]e^{C_o z} \quad (13)$$

where $G_1(-\infty) = C_1 - (1 - g_\omega)A_{11}/C_o^2$.

A complete solution for the second-order stream function in quadrature form can be obtained by using the three known homogeneous solutions[8] of Eq. (8a) and variation of parameters. However, for present purposes it is sufficient to construct the asymptotic form of F_2 from Eqs. (8), (9), (12). Hence,

$$F_2(z \to -\infty) \sim -(g_\omega/2C_o)z^2 - ([g_\omega + A_{21}]/C_o^2)z + (1 - g_\omega)/C_o - (A_{21}\hat{z}/C_o^2) + 0(z^3e^{C_o z}) \quad (14)$$

where A_{21} is an integration constant analogous to A_{11} in Eq. (13). Finally, the asymptotic form of the second-order enthalpy, developed from a series solution of Eq. (8b) is

$$G_2(z \to -\infty) \sim G_2(-\infty) + \left[(1 - g_\omega)g_\omega\left(\frac{a}{6}\right)z^3 + 0(z^2)\right]e^{C_o z} + 0(z^4e^{2C_o z}) \quad (15)$$

where $G_2(-\infty)$ is a constant to be found.

Matching Procedure

The matching form for the outer stream function may be constructed by using the asymptotic results in Eqs. (9, 12, and 14) in Eq. (5a) and then rewriting the expression in terms of η by means of Eq. (2). Then from Eqs. (3a), (4c), and (4d) it follows that

$$-C_o + \beta \ln^2 1/\beta(B_1) + \beta \ln 1/\beta(B_2) + \beta[(g_\omega/C_o^3)e^{C_o\eta} - (g_\omega/2C_o)\eta^2 - (g_w/C_o^2)\eta + B_3] + \ldots \sim -C_o - \beta \ln 1/\beta(A_{11}\eta/C_o^2) + \beta \ln\lambda_o(g_\omega\eta/C_o^2) + \beta \ln 1/\beta \ln\lambda_o(A_{11}/C_o^3) + \beta \ln^2\lambda_o(-g_\omega/2C_o^3) - \beta \ln 1/\beta(A_{11}C_o^2)(\hat{z} - [\ln\lambda_1]/C_o) + \beta \ln\lambda_o[-(g_\omega/C_o^3)\ln\lambda_1 + (g_\omega/C_o^2 + A_{21}/C_o^2)/C_o] + \lambda_o^{-1}(a/\lambda_1)e^{C_o\eta} + \beta\{(-g_\omega/2C_o)\eta^2 + [(g_\omega \ln\lambda_1)/C_o^2 - g_\omega/C_o^2 - A_{21}/C_o^2]\eta - [(g_\omega/2C_o^3)\ln^2\lambda_1 - (g_\omega/C_o^3 + A_{21}/C_o^3)\ln\lambda_1 + A_{21}\hat{z}/C_o^2 - (1 - g_\omega)/C_o]\} + \ldots$$

The key to the remaining matching procedure evolves from the initial observation that the terms growing exponentially in η must be identical. Following a logical progression of term identification, and accounting for self-annihilation due to switchback terms (the second and third terms on the right-hand side must annihilate one another) one finds that

$$\lambda_o = 1/\beta,\ \lambda_1 = aC_o^3/g_\omega \quad (16a)$$

$$A_{11} = g_\omega,\ A_{21} = g_\omega \ln\lambda_1 \quad (16b)$$

$$B_1 = g_\omega/2C_o^3,\ B_2 = (g_\omega/C_o^2)([1 + \ln\lambda_1]/C_o - \hat{z}) \quad (16c)$$

$$B_3 = (g_\omega/C_o^3)[(\ln^2\lambda_1)/2 + (1 - C_o\hat{z})\ln\lambda_1 - C_o^2] + 1/C_o \quad (16d)$$

A similar procedure may be used for the enthalpy expansions; the ultimate results being

$$K_o = K_1 = K_2 = 0,\ K_3 = (1 - g_\omega)g_\omega/C_o^2 \quad (17a)$$

$$G_1(-\infty) = 0,\ G_2(-\infty) = -(1 - g_\omega)g_\omega/C_o^2 \quad (17b)$$

Each of the constants may now be evaluated as functions of g_ω by using the known values of a, C_o and $\hat{z}$.

A careful examination of the matching term leads one to the conclusion that the order of the next terms in the expansions is $\beta^2 \ln^3\beta$.

4. Interpretation of Results

The preceding asymptotic analysis shows that the type of incipient blowoff phenomena considered can be described in terms of a viscous wall layer in which a small adverse pressure gradient is necessary to enforce the condition of zero wall shear, and an outer viscous shear layer. In the former, the stream function and enthalpy are described by

$$f \sim -C_o + \beta \ln^2 1/\beta(g_\omega/2C_o^3) + \beta \ln 1/\beta(g_\omega/C_o^2) \times [(1 + \ln\lambda_1)/C_o - \hat{z}] + \beta[(g_\omega/C_o^3)e^{C_o\eta} - (g_\omega/2C_o)\eta^2 - (g_\omega/C_o^2)\eta + (g_\omega/C_o^3)[(\ln^2\lambda_1)/2] + (1 - C_o\hat{z})\ln\lambda_1 - C_o^2 + 1/C_o] + 0(\beta^2 \ln^3\beta) \quad (18a)$$

$$g \sim g_\omega + \beta(1 - g_\omega)(g_\omega/C_o^2)(e^{C_o\eta} - 1) + 0(\beta^2 \ln^3\beta) \quad (18b)$$

Hence, both the x-wise velocity and the heat transfer are $0(\beta)$ in magnitude. It should be noted that the latter will be positive or negative, depending upon whether the wall (g_ω) is colder or warmer than the external flow ($g = 1$). The stream function form indicates that the perturbation from the critical injection rate C_o must be several orders larger than $0(\beta)$ to be compatible with the given boundary conditions.

The outer shear-layer region, described basically by Lock's mixing layer solution, is observed to be shifted away from the wall by an amount defined by the translational transformation

$$\eta = z + C_o^{-1}[\ln 1/\beta + \ln(a_1C_o^3/g_\omega) + 0(\lambda_2)] \quad (19)$$

Hence, in the η plane the dividing streamline is located at $\eta^* = C_o^{-1}\ln(a_1C_o^3/g_\omega\beta)$ to a first-order approximation showing that the over-all boundary layer thickens logarithmically as $\beta \to 0$. It should be noted, also, that if g_ω is permitted to become small, a further thickening of the boundary layer will occur. Furthermore, in the cold wall limit $g_\omega = 0$ a definite singularity appears. A further elucidation of this matter may be obtained by examining the mass transfer rate formula found from Eqs. (16) and (18a);

$$f(0) \sim -C_o + \beta \ln^2 1/\beta(0.747g_\omega) + \beta \ln 1/\beta(1.49g_\omega) \times (\ln 1/g_\omega - 1.298) + \beta\,(1/C_o + 0.747g_\omega \ln^2 1/g_\omega - 0.872g_\omega \ln 1/g_\omega + 0.358g_\omega) \quad (20)$$

in which one notes the nonconvergent sequence of logarithmic terms in g_ω suggesting that the analysis is not correct for $g_\omega \to 0$. In an effort to ascertain the validity of this remark, an asymptotic solution for Eq. (1) was developed for the special case $g_\omega = 0$. Following a procedure quite similar in philosophy to that used previously, one may find that the stream function and enthalpy in the wall region are described by

$$f \sim -C_o + \beta/C_o + \beta e^{-C_o^2/\beta}f_2(\eta) + \ldots \quad (21a)$$

$$g \sim e^{-C_o^2/\beta}g_1(\eta) + \ldots \quad (21b)$$

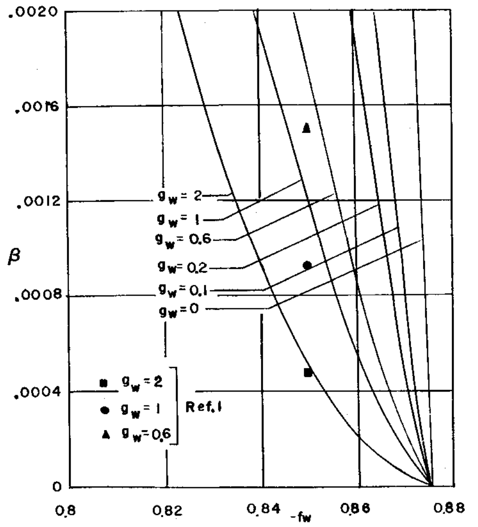

Fig. 1 Variation of pressure gradient parameter for $-f_\omega \to C_o$.

implying transcendentally small gradients. This result is implied by those in Eq. (18) where it should be noted that excluding the β/C_o term in the third-order stream function correction, the algebraic perturbations in β all contain a factor of g_ω. Furthermore, the translational transformation has the form $\eta = z + C_o/\beta + 0(1)$ showing that the boundary-layer thickness for $g_\omega = 0$ is considerably greater than the finite g_ω case. These results indicate quite clearly the reasons for the "pathelogical" numerical results found in Ref. 1. In particular, present numerical schemes cannot deal with such small gradients.[10] Physically, this behavior occurs because the weak pressure gradient is relatively ineffective in turning the cold dense injected fluid.

5. Numerical Results

Numerical results obtained from Eqs. (20), (21) are presented in Figs. 1 and 2 (which are analogous to Figs. 1 and 2 in Ref. 1). In the former, the mass transfer rate $-f_\omega$ is plotted as a function of β for parametric values of g_ω. The latter indicates the corresponding heat-transfer rates. Where possible, Fox and Saland's numerical results have been included.‡ (It should be noted that, in general, the computer calculation cannot deal with the asymptotically small values of β considered in the analysis.) The available comparisons indicate that for a given β the difference between the analytical and numerical values of f_ω is at most a few percent.

The curves in Fig. 1 show a regular progression with decreasing g_ω. The curve for $g_\omega = 0$ was found from Eq. (21a) with the transcendentally small term neglected. The result seems to corroborate the trend shown for the cold wall case in Fig. 1b of Ref. 1 where the uppermost curve has a larger negative slope than the $g_\omega = 0.2$ result at $f_\omega = 0.8$.

A further elucidation of the peculiar behavior of the cold wall case can be developed by considering Fig. 2. Once again the curve found from Eq. (18b) shows a reasonable behavior in the range $0.2 \leq g_\omega < 1.0$. For $g_\omega = 0.1$, one notes a reversal in trend, also found by Fox and Saland. The curve for $g_\omega = 0$, found from Eq. (21b) lies nearly on the abscissa for the scale shown. A comparison with the trend shown in Fig. 2c of Ref. 1 certainly indicates agreement. It would appear then that the peculiar (but not erroneous) results obtained by

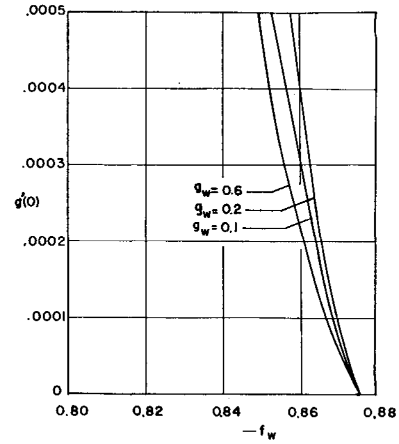

Fig. 2 Variation of heat-transfer parameter for $-f_\omega \to C_o$.

Fox and Saland are explainable in terms of a high inertia (cold dense fluid), weak pressure force interaction.

6. Conclusions

The asymptotic analysis presented here which describes incipient blowoff phenomena associated with zero wall shear verifies the trends found by numerical means in a regime where the latter fails. Of particular interest is the cold wall solution which leads to drastic heat transfer reductions when compared with that for larger values of g_ω.

The analysis also indicates that the essential characteristic dimension of the overall boundary layer in the η plane is of order $\ln 1/\beta$ for finite g_ω, and even thicker, of order $1/\beta$, when $g_\omega = 0$. These values correspond in the physical plane to dimensions of $0(Re^{-1/2} \ln 1/\beta)$ and $0[(Re\beta^2)^{-1/2}]$. Insofar as classical boundary layer theory is concerned, the former involves a rather weak singularity, whereas the latter is considerably stronger. For a given flow (and hence large Re) a sufficiently small pressure gradient implies the failure of classical boundary-layer theory. This difficulty, which appears in several blowoff problems, is discussed in some length in Ref. 5. It is concluded there that a form of analysis similar to that used in the massive blowing studies of Cole and Aroesty[9] must be used.

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‡ Rogers[11] has provided the author with two additional points in Fig. 1 for the case of $g_\omega = 2$ and values of $-f_\omega = 0.86, 0.87$. The numerical values of β, which lie on the $g_\omega = 2$ curve, are $\beta = 2.35 \times 10^{-4}$, and $\beta = 6.09 \times 10^{-5}$, respectively.